Calculation of the Masses and the Running Masses of the Quarks and Leptons from Electroweak to Supersymmetric Grand Unification Mass

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We make a systematic theoretical analysis of the masses of the fermions and their variation with energy by solving the one loop renormalization group equation (RGE) in the Minimal Supersymmetric Standard Model (MSSM). A deceptibly simple common mass for all the fermions around 115 GeV at GUT scale has been found by Deo and Maharana. Here we undertake the unfinished but the important task of calculating the electroweak masses of the fermions at different energies. The proposed parametric unification mass and group theoretic constants for the model are known. The mass of the top quark and its descent is studied by an approximate method very carefully. We find that the Ramond, Roberts and Ross value of the Wolfenstein parameter is reproduced and is nearly equal to 0.22. When raised to integral powers and multiplied by 115 GeV, the whole mass spectra of the remaining eleven fermions are obtained within experimental errors. We deduce formulae for the masses and plot them for all the 12 fermions from $t = log\frac{\mu}{M_z} = 0$ to $t_X = 33$; the GUT mass being $M_X = 2.2 \times 10^{16}$ GeV.

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1. INTRODUCTION

Pati and Salam[1] pioneered the idea that leptons are the fourth colour; quarks and leptons should be brought under the same umbrella of one group so that all forces (except gravity) can be understood in terms of one unifying force parameter near the Planck scale. The six leptons should be treated at par with the six coloured quarks. The elementary constituents of matter would become twelve only. With the availability of enormous data from the high energy accelerators, phenomenological analysis backed by imaginative theories, it has been found that the familiar standard model, which is the product group $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$, can explain all of them successfully. This standard model is characterised by three coupling strengths of the weak, the electromagnetic and the strong interactions. It was conceived that all the three couplings should run, i.e., they change with energy, and eventually become one at the grand unified scale. This could be made possible by solving the relevant RGE. The coefficients of the theory are specified by the group structure alone. Many theoretical models were investigated and only a few years back, it has become essential to bring in supersymmetry. In the minimal version of the MSSM, the beta coefficients are such that they run the coupling strengths to a single value $\frac{4\pi}{g_U^2} = \alpha_{GUT} \simeq \frac{1}{25}$ at a mass of $M_X = 2.2 \times 10^{16}$ GeV. This has been the most attractive result in the current investigations in gauge theories[2].

Next in order, the major challenge in particle physics, was and is the theoretical derivation of the mass spectrum of the quarks and leptons in the same successful MSSM theory. In this model, all the masses of the fermions and the mixing angles were being chosen arbitrarily to account for the 19 free parameters of the theory. In the absence of such a fundamental theory, it has been in vogue to pursue the method which has been known as 'textures analysis'. After finding a suitable texture, one can investigate further and possibly obtain unification mass parameters for all the fermions as a generalisation of the hypothesis by Georgi et al[3]. Eventually, one can predict their individual masses by using the analytical MSSM group coefficients for a RGE for the masses of the fermions. As yet, this has not been successful.

One of the 13 parameters of the standard model was first predicted in 1974 by Gaillard and Lee[4]. Then came the work of the popular mass matrix ansatz of Fritzsch[5]. A complete listing of textures and their relevance to experimental findings were made by Ramond, Roberts and Ross (RRR)[6]. Since they also took the help of RGE, we shall present the essential results of their work in this section. Before that, we present the MSSM Lagrangian

$$\mathcal{L}_M = \bar{Q}_L M_U \Phi_u U_R + \bar{Q}_L M_D \Phi_d D_R + \bar{l}_L M_E \Phi_d E_R + \bar{\nu}_L M_N \Phi_u \nu_R + h.c. \tag{1.1}$$

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We ignore the sparticles and consider the rigid part of the Lagrangian given by Demir [7].

In the renormalisation schemes, the Yukawa couplings $M_F(t)$ and v.e.v. $v_F(t)$ change with energy separately. The Dirac masses are given by

$$m_F(t) = v_F(t)M_F(t) \tag{1.2}$$

The masses considered above are not the masses of the flavor eigenstates of the model.

The one loop RGE, as written by Grzadkowski, Lindner and Theisen [8] for $M_Y(t)$ are

$$16\pi^2 \frac{dM_Y}{dt} = \left(-G_Y + T_Y + S_Y\right) M_Y(t), \tag{1.3}$$

$t = log(\mu/M_Z)$, μ is the renormalisation point and M_Z is the mass of the Z-boson. Here Y=U stands for U-quarks (F=1,2,3): up(u), charm(c) and top(t); Y=D stands for Down quarks (F=4,5,6): bottom(b), strange(s) and down(d); Y=N stands for Neutrinos (F=7,8,9): ν_e, ν_μ, and ν_τ, and Y=E stands for electrons(F=10,11,12): e, μ and τ. $G_Y(t)$ contains the gauge coupling terms, given in Section-2 and

$$T_U = Tr\left(3M_UM_U^\dagger + M_NM_N^\dagger\right), \qquad T_D = Tr\left(3M_DM_D^\dagger + M_EM_E^\dagger\right), \quad T_U = T_N, T_E = T_D, \tag{1.4}$$

and

$$S_U = 3M_UM_U^\dagger + M_DM_D^\dagger, \quad S_D = 3M_DM_D^\dagger + M_UM_U^\dagger, \quad S_E = 3M_EM_E^\dagger + M_NM_N^\dagger, \quad S_N = 3M_NM_N^\dagger + M_EM_E^\dagger. \tag{1.5}$$

To find the couplings, we have to solve the twelve differential equations and determine all the couplings/masses from only one value of coupling at $t = t_X$ or $M_F(M_X)$. First, we turn our attention to the mass of the top quark. Incidentally, Pendleton and Ross [9], Faraggi[10], and some others have predicted the value of the mass of the top quark which was around 175 GeV, even before the top was discovered.

In this paper, all the one loop equations are solved following the same method used for gauge coupling RGE. A particular $M_F(M_Z)$ is obtained relating to $M_F(M_X)$ and other fermions. Considering the top, first we make 'heavy top integral' approximation stated below. Eventhough we aim at single input value $M_F(M_X) = M_U$, which is independent of F, the integrals of the solutions for the masses are such that

$$\int M_{top}^2(\tau)d\tau \gg \int M_{Q\neq top}^2(\tau)d\tau \ggg \int M_{lepton}^2(\tau)d\tau \tag{1.6}$$

Neglect of the integrals other than the top quark, gives a very good result for the $M_{top}(M_X) \simeq M_U$. Following this approximation procedure, we find from other eleven equations that $M_{top}(M_X) \simeq M_F(M_X) \simeq M_U \simeq 115$ GeV for all the fermions.

We choose $M_U \simeq 115$ GeV as the only input. Furthermore, MSSM has two Higgs.

$$\begin{aligned} <\phi_U>= v_u(t) &= v(t)Sin\beta(t) && (1.7)\\ <\phi_D>= v_d(t) &= v(t)Cos\beta(t) && (1.8)\\ v^2(t) &= v_d^2(t) + v_u^2(t) && (1.9)\\ tan\beta(t) &= \frac{v_u(t)}{v_d(t)}. && (1.10) \end{aligned}$$

It is to be noted that $v(t)$ is the vacuum expectation value of single Higgs of the Standard Model. $v(M_Z)$=246 GeV, $Sin\beta_{SM}(M_Z) = \frac{1}{\sqrt{2}}$. We note that $v_0 = \frac{246}{\sqrt{2}}$ GeV=174 GeV, which is close to $M_{top}(M_Z) \cong 175$ GeV. For simplicity, we shall use this as the unit of energy whenever unspecified.

The one loop RGE are

$$\begin{aligned} 16\pi^2 \frac{dv_u}{dt} &= \left(\frac{3}{20}g_1^2 + \frac{3}{4}g_2^2 - Tr(3M_UM_U^\dagger)\right) v_u, && (1.11)\\ 16\pi^2 \frac{dv_d}{dt} &= \left(\frac{3}{20}g_1^2 + \frac{3}{4}g_2^2 - Tr(3M_DM_D^\dagger)\right) v_d. && (1.12) \end{aligned}$$

The calculation of

$$tan\beta(t) = \frac{v_u(t)}{v_d(t)}, \tag{1.13}$$

in MSSM has attracted considerable attention, there exist extensive literature, most of them are given in reference[11]. We note that below the mass scale M_Z of the normal SM, one Higgs v.e.v. satisfies the one loop equation

$$16\pi^2 \frac{dv_{SM}}{dt} = \left(\frac{3}{20}g_1^2 + \frac{3}{4}g_2^2 - Tr(3M_U M_U^\dagger + 3M_D M_D^\dagger)\right) v_{SM}. \tag{1.14}$$

To maintain continuity, we assume that at t=0,

$$tan\beta(M_Z) = tan\beta_{SM}(M_Z) = 1. \tag{1.15}$$

Using the approximation given in equation (1.6), we get from equations (1.11), (1.12) and (1.13),

$$tan\beta(t) \cong exp\left(-\frac{3}{16\pi^2}\int_0^t \left[M_{top}^2(\tau) - M_{bottom}^2(\tau)\right] d\tau\right). \tag{1.16}$$

Using the results given in section-3, equation(3.27), we find that $tan\beta(t)$ is a slowly varying function of t and drops from 1 at 91 GeV to 0.9 at 10^{16} GeV. So the calculation is much simplified if we take $Sin\beta = \frac{1}{\sqrt{2}} = Cos\beta$. This is not ruled out by experiment[8].

The authors (RRR), noted that there are only six possible forms of symmetric mass matrices with three non-zero eigenvalues and three texture zeroes, capable of describing the hierarchy of up or down quark mass matrices. Those are

$$T_1 = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 0 & a_2 & 0 \\ a_2 & b_2 & 0 \\ 0 & 0 & c_2 \end{pmatrix}, \quad T_3 = \begin{pmatrix} a_3 & 0 & 0 \\ 0 & 0 & b_3 \\ 0 & b_3 & c_3 \end{pmatrix},$$
$$T_4 = \begin{pmatrix} 0 & 0 & a_4 \\ 0 & b_4 & 0 \\ a_4 & 0 & c_4 \end{pmatrix}, \quad T_5 = \begin{pmatrix} 0 & a_5 & 0 \\ a_5 & 0 & b_5 \\ 0 & b_5 & c_5 \end{pmatrix} \quad \texttt{and} \quad T_6 = \begin{pmatrix} 0 & a_6 & b_6 \\ a_6 & 0 & 0 \\ b_6 & 0 & c_6 \end{pmatrix}. \tag{1.17}$$

T_5 was the one first pioneered by Fritzsch[5]. More recently Dimopoulos, Hall and Raby[12] have analysed and included the leptons following Georgi[3] and solved the MSSM RGE with some degree of success. RRR tried to analyse all the cases and put them in a CKM matrix form, proposed by Wolfenstein[13] and diagonalise them to the texture types.

$$V_{CKM} = \begin{pmatrix} c_1c_2 - s_1s_2e^{-i\phi} & s_1 + c_1s_2e^{-i\phi} & s_2(s_3 - s_4) \\ -c_1s_2 - s_1e^{-i\phi} & -s_1s_2 + (c_1c_2c_3c_4 + s_1s_4)e^{-i\phi} & s_3 - s_4 \\ s_1(s_3 - s_4) & -c_1(s_3 - s_4) & (c_3c_4 + s_1s_4)e^{i\phi} \end{pmatrix} \tag{1.18}$$
$$= \begin{pmatrix} \lambda^2/2 & \lambda & A\lambda^3(\rho + i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho + i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{1.19}$$

Here $s_i,\ c_i$ (i=1,..,4)are the sines and cosines of mixing angles. From the identities given by Dimopoulos, Hall and Raby[12], following from equations (1.18) and (1.19)

$$\lambda = (s_1^2 + s_2^2 + s_1s_2\ Cos\phi)^{1/2}. \tag{1.20}$$

ϕ is the CKM phase angle and

$$s_1 = \left(\frac{M_d}{M_s}\right)^{\frac{1}{2}} = \lambda, \quad s_2 = \left(\frac{M_u}{M_c}\right)^{\frac{1}{2}} = \lambda^2; \quad s_4 = \left(\frac{M_dM_s}{M_b^2}\right)^{\frac{1}{2}} = \lambda^3. \tag{1.21}$$

As will be discussed later, the $Cos\phi$ defined in [12], we shall get

$$Cos\phi = -\lambda/2, \tag{1.22}$$

and

$$s_3 - s_4 = \lambda^2 A(t) \tag{1.23}$$

where A(t) depends on t.

The small expansion parameter is $\lambda \simeq$ 0.2 and $A \simeq 0.9 \pm 0.1$. Olechowski and Poroski[8] were the first to write down the RG Equations for the parameters

$$16\pi^2 \frac{d|J_{c\rho}|}{dt} = -\ 3\ c\ (h_t^2\ +\ h_b^2)|J_{c\rho}|, \tag{1.24}$$

$$16\pi^2 \frac{dA}{dt} = -\frac{3}{2}\ c\ (h_t^2\ +\ h_b^2)A, \tag{1.25}$$

$$\frac{d\lambda}{dt} = 0, \tag{1.26}$$

$$\frac{d\rho}{dt} = 0, \tag{1.27}$$

$$\frac{d\eta}{dt} = 0. \tag{1.28}$$

Here $c\ =\ 2/3$ for MSSM and $J_{c\rho}$ is the irreducible phase of the CKM matrix. To arrive at equation (1.25), we equate

$$A(t) = \left[\frac{M_{bottom}(t)M_{top}(t)}{M_{top}^2(M_X)}\right]^{-1/7}. \tag{1.29}$$

Using one loop RG equations (1.3) to (1.5), we get

$$A(M_X) = 1.00 \quad \text{and} \quad A(M_Z) = 1.474. \tag{1.30}$$

RRR have made a complete listing and analysis. They arrived at the value λ ==0.22 and the oft quoted result,

$$\begin{aligned} m_\tau\ :\ m_\mu\ :\ m_e\ &=\ m_b\ :\ m_s\ :\ m_d\ =\ 1\ :\ \lambda^2\ :\ \lambda^4 \\ m_t\ :\ m_c\ :\ m_u\ &=\ 1\ :\ \lambda^4\ :\ \lambda^8, \end{aligned} \tag{1.31}$$

which is very well satisfied by experimentally found masses. This has been thought to be like a miracle.

We shall take the masses of the twelve fermions to be real, wherever necessary. In section-2, we shall write the RG Equation with the MSSM coefficients and give the one loop exact solutions. In section-3, the unification mass for all fermions at GUT scale ranging from 113 GeV to 125 GeV as computed by Deo and Maharana[14], will be discussed as a follow up of their letter. An expression for Wolfenstein parameter in terms of RGE coefficients is given in section-4. The reason to raise this parameter by integers n_F to obtain fermion masses except the top, M_F equaling λ^{n_F} multiplied by the parametric unification mass 115 GeV, is given. In sections-5,6 and 7, we suggest an alternative method of calculation of experimental masses of all fermions by a suitable self contained procedure which includes the gauge couplings as well. The equation for the running of all the masses of all the fermions is given in section-8. The results are given in tables and are also shown graphically for greater clarity. The concluding remarks are given in the section-9.

2. RENORMALISATION GROUP EQUATIONS AND SOLUTIONS

As stated, the gauge sector of the standard model is characterised by three coupling constants g_3, g_2 and g_1 of $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$, respectively. However, these couplings are not constants, they change with energy/mass values. The nature of variation is given by the solutions of the RG Equations. The coefficients are calculated by the specific nature of the Standard Model group. For MSSM, the three couplings at mass $M_X = 2.2 \times 10^{16}$ GeV[15] unite to a unified coupling constant $g_U^2/4\pi$=1/24.6. Here, the supersymmetry descends from M_X down to $M_Z \simeq$ 91 GeV, as suggested by Witten. The RG Equations for the couplings in the lowest order are given by

$$16\pi^2 \frac{dg_i(t)}{dt} = c_i g_i^3(t), \quad i = 1, 2, 3. \tag{2.1}$$

The coefficients are c_1 = 6.6, c_2 =1, c_3 = -3. The first two coefficients are positive, indicating that $U_Y(1)$ and $SU_L(2)$ are not asymptotically free, whereas the $SU_C(3)$ colour group is free and makes the entire product group asymptotically free . This implies that in a perturbative formulation, the higher order contributions are small and can be neglected. Therefore, we shall use equation(2.1) only in the gauge sector, with two Higgs[15].

The running parameter t is defined as $t = log_e \frac{\mu}{M_Z}$ so that it varies from 0 to $log_e \frac{M_X}{M_Z} \simeq 33$. The solution to RG Equation(2.1) is

$$\frac{4\pi}{g_i^2(t)} = \frac{4\pi}{g_i^2} - \frac{c_i}{2\pi} t. \tag{2.2}$$

Here, $g_i^2 = g_i^2(0)$ are the coupling strengths in the electroweak scale M_Z. Taking the value of $M_X = 2.2 \times 10^{16}$ GeV and $\frac{4\pi}{g_U^2} = 24.6$, we calculate the values of $\frac{4\pi}{g_1^2} = 59.24$, $\frac{4\pi}{g_2^2} = 29.85$ and $\frac{4\pi}{g_3^2} = 8.85$. These are consistent with the experimental results. Thus the three coupling strengths are descendants of one coupling constant g_U.

Taking the clue from equation(2.1), we rewrite the Yukawa sector SUSY RG Equations given earlier in equation (1.3), which had also been written by Babu[16] following Georgi and Glashow, and Eitchen et al[17] in the following way;

$$\begin{aligned} 16\pi^2 \frac{dM_F(t)}{dt} &= A_F M_F^3(t) + [Y_F(t) - G_F(t)] M_F(t) && (2.3) \\ &= A_F M_F^3(t) + Z_F(t) M_F(t). && (2.4) \end{aligned}$$

The masses are in units of 175 GeV. We repeat for ready reference that the 12 fermions are suffixed as F = 1,2,... 12. F = 1,2,3 are the U-quarks: top(t), charm(c) and up(u). Similarly, F=4,5,6 denote the D-quarks: bottom(b), strange(s) and down(d); F=7,8,9 are the E-leptons: electron(e), muon(μ) and tau(τ); and F=10,11,12 are the N-neutrinoes ν_e, ν_μ, ν_τ. Further, $M_1 = M_{top},\ M_2 = M_{charm},\ \cdots, M_{12} = M_{tau}$.

A_F is a group theoretic factor whose value is '6' for quarks, i.e., F=1,2,$\cdots$,6 and '4' for the leptons i.e., F=7,8,$\cdots$, 12. The positive values indicate the field theory containing Yukawa couplings only and may not be asymptotically free.

Y_F is the mixing term which can be put in matrix form

$$Y_F = \sum_H A_{FH} M_H^\dagger(t) M_H(t), \qquad H = 1, 2, \cdots, 12. \tag{2.5}$$

In MSSM, the matrix A_{FH} is specified by the 144 elements given below,

$$A_{FH} = \begin{pmatrix} 0&3&3&1&0&0&0&0&0&1&1&1 \\ 3&0&3&0&1&0&0&0&0&1&1&1 \\ 3&3&0&0&0&1&0&0&0&1&1&1 \\ 1&0&0&0&3&3&1&1&1&0&0&0 \\ 0&1&0&3&0&3&1&1&1&0&0&0 \\ 0&0&1&3&3&0&1&1&1&0&0&0 \\ 0&0&0&3&3&3&0&1&1&1&0&0 \\ 0&0&0&3&3&3&1&0&1&0&1&0 \\ 0&0&0&3&3&3&1&1&0&0&0&1 \\ 3&3&3&0&0&0&1&0&0&0&1&1 \\ 3&3&3&0&0&0&0&1&0&1&0&1 \\ 3&3&3&0&0&0&0&0&1&1&1&0 \end{pmatrix}. \tag{2.6}$$

The diagonal elements of A_{FH} have been taken out as the cubic term in equation(2.3); so they are zero.

As the model is minimal supersymmetric, the gauge factors $G_F(t)$, which are the sum of gauge couplings, are fixed, we take the values from reference[18] and [12].

$$G_U(t) = \frac{13}{15} g_1^2(t) + 3g_2^2(t) + \frac{16}{3} g_3^2(t) = \sum_{i=1}^{3} K_U^i g_i^2(t). \tag{2.7}$$

F =1,2,3 stand for U and they are degenerate electromagnetic gaugewise. Similarly,

$$\begin{aligned} G_D(t) &= \frac{7}{15} g_1^2(t) + 3g_2^2(t) + \frac{16}{3} g_3^2(t), \qquad F = 4,5,6 && (2.8) \\ G_E(t) &= \frac{9}{5} g_1^2(t) + 3g_2^2(t), \qquad F = 7,8,9 && (2.9) \\ \textit{and} \qquad G_N(t) &= \frac{3}{5} g_1^2(t) + 3g_2^2(t), \qquad F = 10,11,12. && (2.10) \end{aligned}$$

Here $K_N^3 = K_E^3 = 0$, as the leptons do not have the strong colour interaction. We shall need the integrals,

$$-\frac{1}{8\pi^2}\int_0^t d\tau\ G_F(\tau) = \sum_{i=1}^{3}\frac{K_i^F}{c_i}\log(1-\frac{c_i g_i^2 t}{8\pi^2}) \tag{2.11}$$

and

$$-\frac{1}{8\pi^2}\int_0^{t_X} d\tau\ G_F(\tau) \ = \ \sum_{i=1}^{3}\frac{K_i^F}{c_i}\log(1-\frac{c_i g_i^2 t_X}{8\pi^2}) \tag{2.12}$$

$$= \ \sum_{i=1}^{3}\frac{K_i^F}{c_i}\log\frac{g_i^2}{g_U^2}. \tag{2.13}$$

Deo and Maharana[14] made a very important observation that equation(2.4), which has to be solved for a given fermion, does not contain the coeficients of the same mass in the matrix A of equation (2.6). This fact has been overlooked by all previous authors. As a result, the calculational dedails become erroneous and the values obtained are unreliable. The present approach gives a hope of a simple method of entangling the mass due to finding the solution of 12 differententia1 equations. As such, the terms $Z_F(t)$ can be exponiented away. We introduce a subsidiary mass $m_F(t)$, through

$$M_F(t) = m_F(t)\exp\left(\frac{1}{16\pi^2}\int_0^t Z_F(\tau)\ d\tau\right), \tag{2.14}$$

such that $M_F(M_Z) = m_F(M_Z) \equiv m_F(0)$. They satisfy the equation

$$16\pi^2\frac{dm_F(t)}{m_F^3} = A_F\exp\left(\frac{1}{8\pi^2}\int_0^t Z_F(\tau)\ d\tau\right)\ dt. \tag{2.15}$$

They look, astonishingly, similar to the gauge sector one loop RG equation (2.1) and can be solved exactly. Integrating equation(2.15) from M_Z to M_X i.e., from t=0 to t_X, we get

$$\frac{8\pi^2}{m_F^2(M_Z)} = \frac{8\pi^2}{m_F^2(M_X)} + A_F\int_0^{t_X} dt\ \exp\left(\frac{1}{8\pi^2}\int_0^t Z_F(\tau)d\tau\right). \tag{2.16}$$

Putting back the exponential,

$$\frac{M_{top}^2(M_Z)}{M_F^2(M_Z)} = \frac{M_{top}^2(M_Z)}{M_F^2(M_X)}\exp\left(\frac{1}{8\pi^2}\int_0^{t_X} Z_F(\tau)d\tau\right) + \frac{A_F}{8\pi^2}\int_0^{t_X} dt\ \exp\left(\frac{1}{8\pi^2}\int_0^t Z_F(\tau)d\tau\right). \tag{2.17}$$

This is the exact one loop solution. $M_F^2(M_Z)$ is the mass of the fermions at M_Z.

The descent or ascent running of the masses from GUT M_X to electroweak M_Z, can be obtained by integrating equation(2.15) from $t = t_X$ to t. The result which is not given in reference [14] is

$$\frac{8\pi^2 M_{top}^2}{M_F^2(t)} = 8\pi^2\frac{M_{top}^2}{M_F^2(M_X)}\exp\left(\frac{1}{8\pi^2}\int_t^{t_X} Z_F(\tau)d\tau\right) + A_F\int_t^{t_X} dt_1\exp\left(\frac{1}{8\pi^2}\int_t^{t_1} Z_F(\tau)d\tau\right). \tag{2.18}$$

This is also one loop exact. By solving equations (2.17) and (2.18), we can find $M_F(M_X)$ and $M_F(t)$ repsectively.

3. ORIGINAL MASS OF ALL FERMIONS AT M_X ?

The gauge integrals over $G_F(t)$ are easily and acurately calculable. The most difficult task is to evaluate $Y_F(t)$. Even though, it does not contain $M_F(t)$, it is a sum of squares of moduli of the masses of all fermions. For example, from the matrix as given by equation(2.6),

$$Y_{top}(t) = 3M_c^2(t) + 3M_u^2(t) + M_b^2(t) + M_{\nu_e}^2(t) + M_{\nu_\mu}^2(t) + M_{\nu_\tau}^2(t). \tag{3.1}$$

There is mixing of six other fermions for the top.

In the 'top heavy integral' approximation, one retains only those terms containing $M^2_{top}(t)$ occuring in any integral with $Y_F(t)$. Consider the top case. $Y_{top}(t)$ can be set equal to zero in the RG Equation for the top. The top mass is then given by a simple expression

$$1 = \frac{M^2_{top}(M_Z)}{M^2_{top}(M_X)} C_{top} + D_{top}, \tag{3.2}$$

where, using integrals (2.11) to (2.13),

$$C_{top} = \prod_{i=1}^{3} \left(\frac{g_i^2}{g_U^2}\right)^{\frac{K_i^U}{c_i}} = 0.086 \tag{3.3}$$

and (3.4)

$$D_{top} = \frac{6}{8\pi^2} \int_0^{t_X=33} dt \prod_{i=1}^{3} \left(1 - \frac{g_i^2 c_i t}{8\pi^2}\right)^{\frac{K_i^U}{c_i}} = 0.802 \tag{3.5}$$

Putting these values, the original mass of the top, at an energy of 2.2×10^{16} GeV was, approximately,

$$M_{top}(M_X) = M_{top}(M_Z)(1 - D_{top})^{-1/2} C_{top}^{1/2} = 114 \ \text{GeV}. \tag{3.6}$$

This is Deo-Maharana result.

We can write a general formula for all fermions

$$\frac{M^2_{top}(M_Z)}{M^2_F(M_Z)} = \frac{M^2_{top}(M_Z)}{M^2_F(M_X)} C_F + D_F, \tag{3.7}$$

where

$$C_F = \prod_{i=1}^{3} \left(\frac{g_i^2}{g_U^2}\right)^{\frac{K_i^F}{C_i}} \exp\left(\frac{1}{8\pi^2} \int_0^{t_X} Y_F(\tau) d\tau\right) \tag{3.8}$$

$$and \qquad D_F = \frac{A_F}{8\pi^2} \int_0^{t_X} dt \prod_{i=1}^{3} \left(1 - \frac{c_i g_i^2 t}{8\pi^2}\right)^{\frac{K_i^F}{c_i}} \exp\left(\frac{1}{8\pi^2} \int_0^{t} Y_F(\tau) d\tau\right). \tag{3.9}$$

It is not so easy to calculate Y_F for other fermions even with 'heavy top integral' approximation given by equation (1.6). So we consider the Yukawa-like coupling $h_F(t)$ for F=2,3, ..., 12, which is related to $M_F(t)$ as

$$M_F(t) = h_F(t) \exp\left(-\frac{1}{16\pi^2} \int_0^t G_F(\tau) d\tau\right). \tag{3.10}$$

The RG Equation for $h_F(t)$ is,

$$8\pi^2 d\left(\log h_F^2(t)\right) = A_F h_F^2(t) \exp\left(-\frac{1}{8\pi^2} \int_0^t G_F(\tau) d\tau\right) dt + Y_F(t) dt. \tag{3.11}$$

Here $h_F(t)$ contains the usual β-angle factors of the two Higgs system. As has been discussed in Section-1, we may not need this angle in our approach to the problem at hand. If $F > 1$, the heaviest fermion next to the top is the bottom. The first term of equation(3.11), at the M_Z scale, is 1/1235 times smaller, whereas the last term contains one h_{top} in h_{botm}. To a good approximation, and to begin with, we shall neglect this term and express $h_F(t)$ in terms of $Y_F(t)$'s. Then

$$\int_0^{t_X} Y_F(\tau) d\tau = 8\pi^2 \log \frac{h_F^2(M_X)}{h_F^2(M_Z)}. \tag{3.12}$$

Here, $h_F^2(M_Z) = M_F^2(M_Z)$ by definition. In the units of top mass 175 GeV,

$$h_F^2(M_X) = M_F^2(M_X) \exp\left(\frac{1}{8\pi^2} \int_0^{t_X} G_F(\tau) d\tau\right) = M^2_{top}(M_X) \exp\left(\frac{1}{8\pi^2} \int_0^{t_X} G_F(\tau) d\tau\right) \tag{3.13}$$

$$= M^2_{top}(M_Z) = 1. \tag{3.14}$$

So,

$$\int_0^{t_X} Y_F(\tau)d\tau = -8\pi^2 \log M_F^2(M_Z) \tag{3.15}$$

$$or \qquad \exp\left(\frac{1}{8\pi^2}\int_0^{t_X} Y_F(\tau)d\tau\right) = \frac{M_{top}^2(M_Z)}{M_F^2(M_Z)}. \tag{3.16}$$

This is true as long as the first term of equation(3.11) is negligible. Since there is M_F^3 in this term, it is much more justifiable to set $D_{lepton} = 0$. The general equation (3.7) for the leptons gives

$$\frac{M_{top}^2(M_Z)}{M_{lepton}^2(M_Z)} = \frac{M_{top}^2(M_Z)}{M_{lepton}^2(M_X)} C_{lepton}, \tag{3.17}$$

where

$$C_{lepton} = \prod_{i=1}^{3}\left(\frac{g_i^2}{g_U^2}\right)^{\frac{K_i^{lepton}}{C_i}},$$

$$M_{lepton}^2(M_X) = M_{lepton}^2 C_{lepton} \tag{3.18}$$

$$\text{or} \qquad M_{lepton}(M_X) = M_{lepton}^{expt.} C_{lepton}^{1/2}. \tag{3.19}$$

Putting the gauge constants and masses in the above, as experimentally reported, we get the masses at GUT scale for different leptons as given in Table-I,

TABLE I: Unification scale mass for leptons

Lepton	Unification scale mass (GeV)
e	116
μ	116
τ	116
ν_e	126
ν_μ	126
ν_τ	126

At the grand unification mass, the electron, the muon and the tau climb to 116 GeV in this approximation. The calculation for the neutrinoes are not reliable as the isospin factors are uncertain and may be inaccurate.

We are now left with the five quarks. For them, we attempt to find the next leading order approximation, i.e., we first set the first term equal to zero and obtain

$$\exp\left(\frac{1}{8\pi^2}\int_0^{t_X} Y_Q(\tau)d\tau\right) = \frac{1}{h_Q^2(M_Z)} = \frac{M_{top}^2(M_Z)}{M_Q^2(M_Z)}. \tag{3.20}$$

This is used in the calculation for C_Q of equation(3.8), which gives

$$C_Q = \prod_{i=1}^{2}\left(\frac{g_i^2}{g_U^2}\right)^{\frac{K_i^Q}{C_i}} \frac{1}{M_Q^2(M_Z)}. \tag{3.21}$$

Using

$$\exp\left(\frac{1}{8\pi^2}\int_0^{t} Y_F(\tau)d\tau\right) = \frac{h_Q^2(t)}{M_Q^2(M_Z)}, \tag{3.22}$$

in equation(3.9), we get

$$D_Q = \frac{6}{8\pi^2}\int_0^{t_X} \frac{h_Q^2(t)}{M_Q^2(M_Z)} \prod_{i=1}^{2}\left(1-\frac{g_i^2 c_i t}{8\pi^2}\right)^{\frac{K_i^Q}{c_i}} dt. \tag{3.23}$$

We look for ways to calculate $h_Q(t)$. We can retain only the top quark in the RG equation in a slightly different way than what has been taken by RRR[6] and get,

$$8\pi^2 d\log h_Q^2(t) \simeq N_Q dt. \tag{3.24}$$

We have

$$N_{charm} = 3, \quad N_{up} = 3, \quad N_{bottom} = 1, \quad N_{strange} = N_{down} = 0. \tag{3.25}$$

To use this value in Y_F, it is necessary to take an average as the couplings change very rapidly,

$$\overline{\log h_Q^2(t)} = \frac{1}{t_X}\int_0^t \frac{d}{d\tau}\log h_Q(\tau)d\tau = \frac{1}{t_X}\int_0^t \frac{N_Q}{8\pi^2}d\tau = \frac{t}{t_X}\frac{N_Q}{8\pi^2}, \tag{3.26}$$

or

$$\overline{h_Q^2(t)} = \exp\left(\frac{t}{t_X}\frac{N_Q}{8\pi^2}\right). \tag{3.27}$$

This expression for Yukawa coupling, is not much different from unity for all allowed t's. We arrive at the following result,

$$D_Q = \frac{6}{8\pi^2}\int_0^{t_X} dt \exp\left(\frac{t}{t_X}\frac{N_Q}{8\pi^2}\right)\prod_{i=1}^{3}\left(1-\frac{g_i^2 c_i t}{8\pi^2}\right)^{\frac{K_i^Q}{c_i}}. \tag{3.28}$$

The unification mass is calculated numerically from

$$M_Q(M_X) = M_{top}\left(\frac{C_Q}{1-D_Q}\right)^{1/2}. \tag{3.29}$$

The results, for the quarks, are given in Table-II. We note that $M_{charm}(M_X) = M_{up}(M_X) \neq M_{top}(M_X)$ and $M_{strange}(M_X) = M_{down}(M_X) \neq M_{bottom}(M_X)$.

TABLE II: Unification scale mass for quarks

Quark	Unification scale mass (GeV)
Top	114
Charm	115
Up	115
Bottom	119
Strange	118
Down	118

Thus, we have shown that all fermions seem to originate at an energy 2.2×10^{16} GeV with equal mass of about 115 GeV. Perhaps, this is due to the equality of A$\simeq$ 1 in Wolfenstein's parametrisation of CKM matrix. In this perturbative method of solution for finding the unification mass, information about the masses of the 11 fermions has been lost due to cancellation of $M^2(M_Z)$ in both r.h.s. and l.h.s. of the equation (2.17) due to use of equations (3.16) and (3.22). The result of a common mass at the origin is atleast a hypothesis and has an approximation [14].

The top mass decreases with energy but all the other quarks and leptons, starting from the value at M_X acquire smaller and smaller values and become quite light in the electroweak scale. The descent or ascent equation(2.18) describing the 'run' can be put in a form like equation(3.7).

$$\frac{M_{top}^2(M_Z)}{M_F^2(t)} = \frac{M_{top}^2(M_Z)}{M_F^2(M_X)}C_F(t) + D_F(t), \tag{3.30}$$

where

$$C_F(t) = a_F(t)\exp\left(\frac{1}{8\pi^2}\int_0^{t_X} Y_F(\tau)d\tau\right), \tag{3.31}$$

$$D_F(t) = \frac{A_F}{8\pi^2}\int_t^{t_X} dt_1 b_F(t_1)\exp\left(\frac{1}{8\pi^2}\int_t^{t_1} Y_F(\tau)d\tau\right), \tag{3.32}$$

$$a_F(t) = \prod_{i=1}^{3}\left[\frac{\left(1-\frac{g_i^2 c_i t_X}{8\pi^2}\right)}{\left(1-\frac{g_i^2 c_i t}{8\pi^2}\right)}\right]^{\frac{K_i^F}{c_i}}, \tag{3.33}$$

$$and \quad b_F(t) = \prod_{i=1}^{3}\left[\frac{\left(1-\frac{g_i^2 c_i t_1}{8\pi^2}\right)}{\left(1-\frac{g_i^2 c_i t}{8\pi^2}\right)}\right]^{\frac{K_i^F}{c_i}}. \tag{3.34}$$

For the top, we can take $Y_F \to 0$ and calculate variation of its mass from $M_U \simeq 115$ GeV to the top mass 175 GeV.

$$M_{top}(t) = \frac{M_{top}(M_X)}{\left(C_{top}(t) + \frac{M_U^2}{M_{top}^2}D_{top}(t)\right)^{1/2}} \tag{3.35}$$

The values are given in Table-V of Section-8.

For other cases, we shall try to fit them into a scheme which is not only much simpler, but has better physical content. However, in the following section, we discuss a different route for solutions without specifying gauge factors completely.

4. DEDUCTION OF WOLFENSTEIN PARAMETERS AND THE ROTATIONAL INTEGERS

Let us construct a function $B_F(t)$ such that

$$8\pi^2\frac{d}{dt}\log B_F^2(t) = Z_F(t). \tag{4.1}$$

Then

$$\exp\left(\frac{1}{8\pi^2}\int_0^{t_X} Z_F(\tau)d\tau\right) = \frac{B_F^2(M_X)}{B_F^2(M_Z)}, \tag{4.2}$$

$$\exp\left(\frac{1}{8\pi^2}\int_{t_X}^{0} Z_F(\tau)d\tau\right) = \frac{B_F^2(M_Z)}{B_F^2(M_X)}, \tag{4.3}$$

$$\exp\left(\frac{1}{8\pi^2}\int_0^{t} Z_F(\tau)d\tau\right) = \frac{B_F^2(t)}{B_F^2(M_Z)}, \tag{4.4}$$

$$\text{and}\exp\left(\frac{1}{8\pi^2}\int_{t_X}^{t} Z_F(\tau)d\tau\right) = \frac{B_F^2(t)}{B_F^2(M_X)}. \tag{4.5}$$

Equation(2.17) reduces to

$$M_F(M_Z) = \frac{\frac{B_F(M_Z)}{B_F(M_X)}M_F(M_X)}{\left(1+\frac{M_F^2(M_X)}{M_{top}^2}\frac{A_F}{8\pi^2}\int_0^{t_X}dt\frac{B_F^2(t)}{B_F^2(M_Z)}\right)^{1/2}}. \tag{4.6}$$

As indicated in the last section, except the top, we neglect the terms with A_F. Then

$$M_F(M_Z) \simeq M_F(M_X)\frac{B_F(M_Z)}{B_F(M_X)} = M_F(M_X)\exp\left(-\frac{I_F}{16\pi^2}\right) = M_F(M_X)\lambda^{n_F}, \tag{4.7}$$

where the integral I_F is

$$I_F = \int_0^{t_X} Z_F(t)dt = \int_0^{t_X} \left(\sum_G A_{FG} M_G^\dagger(t) M_G(t) - G_F(t) \right) dt, \tag{4.8}$$

with A_{1G}=0 to indicate that this is not for the top. Since the ratio of the two fermion masses can be put as $\frac{M_{F_1}}{M_{F_2}} = \lambda$, the parameter M_ν cancels out and λ is the Wolfenstein parameter. The purpose of introducing the logarithms is to incorporate the possible multivaluedness of I_F and obtain the integral powers of λ. From equation (4.8),

$$\begin{aligned}
I_F &= \int_0^{t_X} Z_F(t)dt = \int_0^{t_X} \left(\sum_G A_{FG} M_G^\dagger(t) M_G(t) - G_F(t) \right) dt && (4.9)\\
&= \frac{1}{2} \int_0^{t_X} \left(\sum_G A_{FG} M_G^\dagger(t) M_G(t) - G_F(t) + \sum_G A_{FG} M_G^\dagger(-t) M_G(-t) - G_F(-t) \right) dt && (4.10)\\
&= \frac{1}{4} \int_{-t_X}^{t_X} \frac{dt}{dM_F} dM_F(t) \left(\sum_G A_{FG} M_G^\dagger(t) M_G(t) - G_F(t) + \sum_G A_{FG} M_G^\dagger(-t) M_G(-t) - G_F(-t) \right) && (4.11)\\
&= \frac{16\pi^2}{4} \int_{-M_U}^{M_U} \frac{dM_F(t)}{M_F(t)} \frac{\left(\sum_G A_{FG} M_G^\dagger(t) M_G(t) - \frac{1}{2}(G_F(t) + G_F(-t)) \right)}{\left(A_F M_F^\dagger(t) M_F(t) - G_F(t) \right)}, && (4.12)
\end{aligned}$$

where we have used equation (2.4). Let us set

$$M_F(t) = M_U e^{i\theta_F(M_F(t))n_F} = M_U e^{i\theta_F(t)n_F},$$

where n_F is an integer and we will call it rotational integer. Using the above in (4.12), we get

$$I_F = i n_F \frac{16\pi^2}{2} \int_{-t_X}^{t_X} d\theta_F(t) \frac{\sum_G A_{FG}}{A_F + \sum_G A_{FG} - \frac{G_F(t)}{M_U^2}}. \tag{4.13}$$

We have omitted inconsequential factor $\frac{G_F}{M_U^2}$ in the numerator. We note that for quarks $A_F + \sum_{G=1}^{6}$=6+7=13 and for leptons 4+9=13. The integral is almost a constant and isolate the integer n_F, characterising F from the inegral. Using $1 = \frac{1}{12}\sum_H$ in equation (4.13), we have

$$I_F = i n_F \frac{16\pi^2}{2} \frac{1}{12} \sum_H \sum_G A_{HG} \int_{-t_X}^{t_X} d\theta_H(t) \frac{1}{A_H + \sum_G A_{HG} - \frac{G_H}{M_U^2}}. \tag{4.14}$$

In the above we have averaged over the twelve fermions. Retracing the steps and using $M_H(t) = M_U e^{i\theta_H(t) M_U^2}$ in equation(4.14), we finally get

$$I_F \simeq n_F \frac{1}{12} \sum_H \sum_G A_{HG} \int_0^{t_X} dt = \frac{1}{12} n_F t_X \sum_H \sum_G A_{HG}. \tag{4.15}$$

First, we shall be interested in the CKM matrix for quarks with lepton masses taken as zero. Then let G vary from 1 to 6, whereas F will be taking values from 2 to 12, since all of them contains 6 quarks. Only the coefficients A_{HG} are needed to calculate I_F of equation (4.15). The coefficients A_{FG} have non-vanishing values for F=2,3,$\cdots$,12 and G=1,2,$\cdots$,6.

$$\text{For} \quad F = 2, \cdots, 6: \qquad A_{F1} + A_{F2} + \cdots + A_{F6} = 7, \tag{4.16}$$

and

$$\text{For } F = 7, \cdots, 12: \qquad A_{F1} + A_{F2} + \cdots + A_{F6} = 9. \tag{4.17}$$

For the twelve fermions, the sum of the values of the coefficients A_{FG} is $(7 \times 5) + (9 \times 6) = 89$. The average of Z, i.e., $\bar{Z}$ is geven

$$\bar{Z} = 89/12, \qquad \text{and} \qquad I_F = n_F\ t_X\ \frac{89}{12}. \tag{4.18}$$

From equation(4.7), the Wolfenstein parameter λ turns out to be

$$\lambda = \exp\left(-\frac{t_X}{16\pi^2}\frac{89}{12}\right) = 0.219 \tag{4.19}$$

This is an excellent result in spite of the approximate estimates. The masses of all the fermions due to quark-lepton equivalence other than the top is

$$M_F(M_Z) \simeq M_F(M_X)\lambda^{n_F} \simeq \lambda^{n_F} M_U \tag{4.20}$$

The table-III identifies the particles. We have increased n_F by neighbourhood integers which we have called the rotational integers.

TABLE III: Identification of fermions

n_F	Mass in GeV	Fermion
2	5.5	bottom(b)
3	1.2	charm(c),tau lepton(τ)
4	0.264	strange(s), muon(μ)
6	0.012	down(d),tau neutrino(ν_τ)
7	0.0028	up(u)
8	6×10^{-4}	electron(e)
9	1.33×10^{-4}	muon neutrino(ν_μ)
16	3×10^{-9}	electron neutrino(ν_e)

5. GAUGE CONTRIBUTIONS

5.1. The top mass

It is easy to write down the equation for the top mass in terms of the mass $M_F(M_X)$ and gauge couplings.

$$M_{top} = M_{top}(M_X)\left(\frac{1-d_{top}}{a_{top}}\right)^{1/2}. \tag{5.1}$$

We shall also need the equation for the descent of top mass

$$\frac{M_{top}^2}{M_{top}^2(t)} = \frac{M_{top}^2}{M_{top}^2(M_X)}a_{top}(t) + D_{top}(t). \tag{5.2}$$

The equation contains only the gauge factors.

5.2. The masses of charm and up quarks.

The RG Equation for the top is non-linear and intermixed. Specifically it is

$$8\pi^2\frac{d}{dt}\log M_{top}^2(t) = -G_U + 6M_{top}^2(t) + Y_{top}(t). \tag{5.3}$$

Following a texture analysis of the RG solutions, we introduce a function $B(t)$,(not to be confused with $B_F(t)$), which satisfies

$$8\pi^2\frac{d}{dt}\log B^2(t) = -G_U + 6M_{top}^2(t). \tag{5.4}$$

Equation(5.4) does not specify the function $B(t)$ completely except that $d\log B(t) = d\log(M_{top}(t))$. This lone restriction gives us an infinite number of free choices. Because letting B$\rightarrow \xi$B, where ξ is an arbitrary constant, does

not change the top mass function $M_{top}(t)$. For simplicity, we take the second derivative of this function $B(t)$ to be zero so that

$$d\log B(t) = C_B dt. \tag{5.5}$$

C_B is essentially $d\log M_{top}(t)$, $M_{top}(t)$ changes by only 30 to 50 GeV as the mass μ goes from 91 GeV to 2.2×10^{16} GeV. $C_B \simeq (175 - (123 \quad \text{to} \quad 115))/175 \simeq 0.297 \quad \text{to} \quad 0.342$. We take $C_B = 0.3$. The nonequiness due to nonlinearity can also be seen as follows:

$$dlogB(t) = dlogM(t) = dlog\frac{M_{top}}{M_o} = \frac{dt}{16\pi^2}\left[-G_U + 6\left(\frac{M_{top}}{M_o}\right)^2\right].$$

M_o is an arbitrary constant. We can choose M_o suitably so that

$$\left[-G_U + 6\left(\frac{M_{top}}{M_o}\right)^2\right]/16\pi^2 \simeq 0.3 = C_B.$$

Integrating from $t = t_X(M_X)$ to $t = 0(M_Z)$, we have

$$\log\frac{B_Z}{B_U} = -C_B t_X, \tag{5.6}$$

and

$$\frac{B_Z}{B_U} = e^{-C_B t_X} = (0.192)^6. \tag{5.7}$$

Furthermore, integrating from $t = t_X$ to arbitrary t,

$$\log\frac{B(t)}{B_U} = C_B(t - t_X), \tag{5.8}$$

or

$$\frac{B(t)}{B_U} = \exp\left(C_B(t - t_X)\right) = \left[\left(\frac{B_Z}{B_u}\right)^{1/6}\right]^{(1-t/t_X)}. \tag{5.9}$$

Let us examine the case for the charm. We have

$$\begin{aligned}
8\pi^2\frac{d}{dt}\log M_c^2(t) &= -G_U + 3M_t^2, \\
&= -G_U + \frac{1}{2}[6M_t^2 - G_U] + \frac{1}{2}G_U, \\
&= -\frac{1}{2}G_U + 4\pi^2\frac{d}{dt}\log B^2(t).
\end{aligned} \tag{5.10}$$

Integrating from t_X to t,

$$\log\frac{M_{charm}^2(t)}{M_{charm}^2(M_X)} = \frac{1}{2}\log\frac{B^2(t)}{B_U^2} - \frac{1}{16\pi^2}\int_{t_X}^{t} G_U(t)dt, \tag{5.11}$$

$$M_{charm}(t) = M_U\left(\frac{B(t)}{B_U}\right)^{1/2}\exp\left(-\frac{1}{32\pi^2}\int_{t_X}^{t} G_U(\tau)d\tau\right), \tag{5.12}$$

$$M_{charm}(M_Z) = M_U\left(\frac{B_Z}{B_U}\right)^{1/2} C_{top}^{-1/4}, \qquad C_{top}^{-1/4} = 1.8466, \tag{5.13}$$

$$and \quad M_c = \left[\left(\frac{B_Z}{B_U}\right)^{1/6} C_{top}^{-1/12}\right]^3 M_{charm}(M_X). \tag{5.14}$$

From section-4, we now calculate

$$\lambda_{charm} = e^{-c_B t_X/6} C_{top}^{-1/12} = 0.221 \tag{5.15}$$

Again this is a good result. The mass of the charm is 1.24 GeV from the equation (5.14) in very good agreement with the experimental value. The rotational integer n_F is three as in Table-III.

We continue further and consider the up quark. The 'heavy top integral' approximation of the RGE for the up quark is

$$8\pi^2 \frac{d}{dt} \log M_{up}^2 = -G_U + 3M_{top}^2(t). \tag{5.16}$$

Guessing from the general rotational integral parametrization, as shown in Table-III as λ^7, we write this equation as

$$\begin{aligned} 8\pi^2 \frac{d}{dt} \log M_{up}^2 &= -G_U + \frac{1}{2}(6M_{top}^2(t)), \\ &= -\frac{G_U}{2} + 8\pi^2 \frac{d}{dt} \log B^2(t) - \frac{1}{2} 8\pi^2 \frac{d}{dt} \log M_{top}^2(t). \end{aligned} \tag{5.17}$$

Integrating, we get

$$\frac{M_{up}}{M_U} = \left(\frac{B_Z}{B_u}\right)\left(\frac{M_U}{M_{top}}\right)^{1/2} a_U^{-1/4}, \tag{5.18}$$

$$\lambda_{up} = \left[\left(\frac{B_Z}{B_u}\right)\left(\frac{M_U}{M_{top}}\right)^{1/2} a_U^{-1/4}\right]^{1/7} = 0.220. \tag{5.19}$$

This gives $M_{up} \simeq 0.0029$ GeV, with $M_U = M_{top}(M_X)$, a little lower value but quite close to the experimental result. The values depend crucially on C_B, which is itself not exact. It is also true that M_U may be a little different from 115 GeV.

6. MASSES OF THE DOWN QUARKS

The calculation of the mass of up-quark has set the method of finding solutions close to the experimental values by using the texture analysis function $B(t)$ ignoring the terms with coefficients A_F. We present a general recipe from the 'heavy top integral' approximation. Let the RG Equation for any $F \neq 1$ be

$$8\pi^2 \frac{d}{dt} \log M_F^2 = -G_F + N_F M_{top}^2. \tag{6.1}$$

Here N_F can be zero as well.

$$\begin{aligned} N_F M_{top}^2(t) &= \frac{N_F}{6}\left(6M_{top}^2(t) - G_U\right) + \frac{N_F}{6} G_U, \\ &= \frac{N_F + \eta_F}{6}\left(8\pi^2 \frac{d}{dt} \log B^2(t)\right) - \frac{\eta_F}{6}\left(8\pi^2 \frac{d}{dt} \log M_{top}^2(t)\right) + \frac{N_F}{6} G_U. \end{aligned} \tag{6.2}$$

η_F can be any arbitrary coefficient. All of them will satisfy the RG Equation in the 'heavy top integral' approximation. But they should let n_F and λ be such that they are within rotational integers and gauge interaction contributions.

Integrating from the known values, t_X to 0, we get

$$M_F(M_Z) = M_U \exp\left(\frac{1}{16\pi^2}\int_0^{t_X} G_F(t)\ dt - \frac{N_F}{96\pi^2}\int_0^{t_X} G_U(t)\ dt\right)\left(\frac{B_Z}{B_U}\right)^{\frac{N_F+\eta_F}{6}}\left(\frac{M_U}{M_{top}}\right)^{\frac{\eta_F}{6}}. \tag{6.3}$$

For the down quarks,

$$\begin{aligned}
8\pi^2 \frac{d}{dt} \log M_b^2 &= -G_D + M_{top}^2, &(6.4)\\
8\pi^2 \frac{d}{dt} \log M_s^2 &= -G_D, &(6.5)\\
8\pi^2 \frac{d}{dt} \log M_d^2 &= -G_D, &(6.6)\\
\text{So} \qquad 8\pi^2 \frac{d}{dt} \log M_b^2 &= -G_D + 2M_{top}^2 - M_{top}^2, &(6.7)\\
&= -G_D + \frac{2.5}{6}(6M_{top}^2 - G_U) - \frac{1.5}{6}(6M_{top}^2 - G_U) + \frac{G_U}{6}, &(6.8)\\
&= -G_D + \frac{2.5}{6}(8\pi^2 \frac{d}{dt} \log B^2) - \frac{1.5}{6}(8\pi^2 \frac{d}{dt} \log M_{top}^2) + \frac{G_U}{6}. &(6.9)
\end{aligned}$$

Integrating from t_X to 0, we get

$$\begin{aligned}
\frac{M_b}{M_U} &= \left(\frac{B_Z}{B_U}\right)^{2.5/6} \left(\frac{M_U}{M_{top}}\right)^{1.5/6} a_D^{-1/2} a_U^{1/12}, &(6.10)\\
&= \left[\left(\frac{B_Z}{B_U}\right)^{2.5/12} \left(\frac{M_U}{M_{top}}\right)^{1.5/12} a_D^{-1/4} a_U^{1/24}\right]^2. &(6.11)
\end{aligned}$$

The quantity in the square bracket is $\lambda \simeq 0.203$. This gives the value of the bottom mass as to be 4.7 GeV.

For the strange we have,

$$8\pi^2 \frac{d}{dt} \log M_s^2 = -G_D + \frac{4.5}{6}\left(6\pi^2 \frac{d}{dt} \log B^2 - G_U\right) - \frac{4.5}{6}\left(6\pi^2 \frac{d}{dt} \log M_{top}^2 - G_U\right). \tag{6.12}$$

This leads to

$$\frac{M_s}{M_U} = \left(\frac{B_Z}{B_U}\right)^{4.5/6} \left(\frac{M_U}{M_{top}}\right)^{4.5/6} a_D^{-1/2} = \left[\left(\frac{B_Z}{B_U}\right)^{4.5/24} \left(\frac{M_U}{M_{top}}\right)^{4.5/24} a_D^{-1/8}\right]^4 = \lambda^4. \tag{6.13}$$

Here λ comes out to be near 0.196 and the strange mass is found to be 0.168 GeV.

Proceeding further to the down quark, we get

$$8\pi^2 \frac{d}{dt} \log M_d^2 = -G_D + \frac{6.5}{6}\left(6\pi^2 \frac{d}{dt} \log B^2 - G_U\right) - \frac{6.5}{6}\left(6\pi^2 \frac{d}{dt} \log M_{top}^2 - G_U\right), \tag{6.14}$$

which gives

$$\frac{M_d}{M_U} = \left(\frac{B_Z}{B_U}\right)^{6.5/6} \left(\frac{M_U}{M_{top}}\right)^{6.5/6} a_D^{-1/2} = \left[\left(\frac{B_Z}{B_U}\right)^{6.5/36} \left(\frac{M_U}{M_{top}}\right)^{6.5/36} a_D^{-1/12}\right]^6 = \lambda^6. \tag{6.15}$$

This gives $\lambda \simeq 0.19$ and M_s=0.0053 GeV. The ratio $M_s/M_d \simeq 31$. However, from equation(1.31), this ratio is

$$\frac{M_s}{M_d} = \lambda^{-2} = (0.2)^{-2} = 25. \tag{6.16}$$

These can be considered as gauge interaction corrections to the Wolfenstein parameter.

7. THE MASSES OF THE LEPTONS

Ignoring the non-linear terms proportional to A_F=4, the RG Equation, retaining the top quark only, for the six leptons are

$$8\pi^2 \frac{d}{dt}\log M_e^2 = -G_E, \tag{7.1}$$

$$8\pi^2 \frac{d}{dt}\log M_\mu^2 = -G_E, \tag{7.2}$$

$$8\pi^2 \frac{d}{dt}\log M_\tau^2 = -G_E, \tag{7.3}$$

$$8\pi^2 \frac{d}{dt}\log M_{\nu_e}^2 = -G_N + 3M_{top}^2, \tag{7.4}$$

$$8\pi^2 \frac{d}{dt}\log M_{\nu_\mu}^2 = -G_N + 3M_{top}^2, \tag{7.5}$$

$$8\pi^2 \frac{d}{dt}\log M_{\nu_\tau}^2 = -G_N + 3M_{top}^2. \tag{7.6}$$

For the first three electron-leptons, we write, with choices based on $\lambda \simeq .22$ and rotational integer n_F,

$$8\pi^2 \frac{d}{dt}\log M_e^2 = -G_E + \frac{30}{24}\left(8\pi^2 \frac{d}{dt}\log B^2(t) - G_U\right) - \frac{30}{24}\left(8\pi^2 \frac{d}{dt}\log M_{top}^2(t) - G_U\right), \tag{7.7}$$

$$8\pi^2 \frac{d}{dt}\log M_\mu^2 = -G_E + \frac{17}{24}\left(8\pi^2 \frac{d}{dt}\log B^2(t) - G_U\right) - \frac{17}{24}\left(8\pi^2 \frac{d}{dt}\log M_{top}^2(t) - G_U\right), \tag{7.8}$$

$$8\pi^2 \frac{d}{dt}\log M_\tau^2 = -G_E + \frac{11}{24}\left(8\pi^2 \frac{d}{dt}\log B^2(t) - G_U\right) - \frac{11}{24}\left(8\pi^2 \frac{d}{dt}\log M_{top}^2(t) - G_U\right), \tag{7.9}$$

and obtain

$$\frac{M_e}{M_U} = \left(\frac{B_Z}{B_U}\right)^{5/4}\left(\frac{M_U}{M_{top}}\right)^{5/4} a_E^{-1/2} = \left[\left(\frac{B_Z}{B_U}\right)^{5/32}\left(\frac{M_U}{M_{top}}\right)^{5/32} a_E^{-1/16}\right]^8 = \lambda_e^8, \tag{7.10}$$

which gives $\lambda_e = 0.209$; $M_e = 4\times 10^{-4}$ GeV. Similarly

$$\frac{M_\mu}{M_U} = \left(\frac{B_Z}{B_U}\right)^{17/24}\left(\frac{M_U}{M_{top}}\right)^{17/24} a_E^{-1/2} = \left[\left(\frac{B_Z}{B_U}\right)^{17/120}\left(\frac{M_U}{M_{top}}\right)^{17/120} a_E^{-1/10}\right]^5 = \lambda_\mu^5, \tag{7.11}$$

which gives $\lambda_\mu = 0.238$; $M_\mu = 0.09$ GeV,
and

$$\frac{M_\tau}{M_U} = \left(\frac{B_Z}{B_U}\right)^{11/24}\left(\frac{M_U}{M_{top}}\right)^{11/24} a_E^{-1/2} = \left[\left(\frac{B_Z}{B_U}\right)^{11/72}\left(\frac{M_U}{M_{top}}\right)^{11/72} a_E^{-1/6}\right]^3 = \lambda_\tau^3, \tag{7.12}$$

which gives $\lambda_\tau = 0.23$; $M_\tau = 1.36$ GeV.

The masses of the neutrinoes have not yet been measured, only limits have been set. The reported values, as shown in Table-III, fall into a good pattern, namely, $M_{\nu_e} = M_U\lambda^{16}$, $M_{\nu_\mu} = M_U\lambda^9$ and $M_{\nu_\tau} = M_U\lambda^6$, λ=0.22. So the equivalent, degeneracy lifting equations, showing the choices of n_F are

$$\begin{aligned} 8\pi^2 \frac{d}{dt}\log M_{\nu_e}^2 &= -G_N + \frac{5}{2}\left(6M_{top}^2 - G_U\right) - 2\left(6M_{top}^2 - G_U\right) + \frac{1}{2}G_U, \\ &= -G_N + \frac{5}{2}\left(8\pi^2\frac{d}{dt}\log B^2\right) - 2\left(8\pi^2\frac{d}{dt}\log M_{top}^2\right) + \frac{1}{2}G_U, \end{aligned} \tag{7.13}$$

$$8\pi^2 \frac{d}{dt}\log M_{\nu_\mu}^2 = -G_N + \frac{3}{2}\left(8\pi^2\frac{d}{dt}\log B^2\right) - \left(8\pi^2\frac{d}{dt}\log M_{top}^2\right) + \frac{1}{2}G_U, \tag{7.14}$$

$$8\pi^2 \frac{d}{dt}\log M_{\nu_\tau}^2 = -G_N + \left(8\pi^2\frac{d}{dt}\log B^2\right) - \frac{1}{2}\left(8\pi^2\frac{d}{dt}\log M_{top}^2\right) + \frac{1}{2}G_U. \tag{7.15}$$

The neutrino masses are obtained as

$$M_{\nu_e} = M_U a_N^{-1/2} a_U^{1/4} \left(\frac{B_Z}{B_U}\right)^{5/2} \left(\frac{M_U}{M_{top}}\right)^2 \simeq \lambda_{\nu_e}{}^{16} \simeq 3 \times 10^{-9}\texttt{GeV}, \tag{7.16}$$

$$M_{\nu_\mu} = M_U a_N^{-1/2} a_U^{1/4} \left(\frac{B_Z}{B_U}\right)^{3/2} \left(\frac{M_U}{M_{top}}\right) \simeq \lambda_{\nu_\mu}{}^{9} \simeq 1.4 \times 10^{-4}\texttt{GeV}, \tag{7.17}$$

$$M_{\nu_\tau} = M_U a_N^{-1/2} a_U^{1/4} \left(\frac{B_Z}{B_U}\right) \left(\frac{M_U}{M_{top}}\right)^{1/2} \simeq \lambda_{\nu_\tau}{}^{6} \simeq 1.3 \times 10^{-2}\texttt{GeV}. \tag{7.18}$$

This completes our calculation of the masses of the quarks and leptons in terms of known and calculable quantities η_F, satisfying the RG Equations. It is absolutely necessary to write down the solutions of the RG Equations for each quark and leptons separately, then alone, the contributions from the gauge interactions and the Yukawa mass coefficients A_{FH} can be ascertained.

The values of the coefficients η_F used above have been calculated, upto to the nearest fraction, from the cited rotational integers n_F in Table-III and the top coupling coefficients N_F from the RG Equations. They are given in the Table-IV.

TABLE IV: Coefficients of η_F for fermions

quarks	n_F	N_F	η_F	leptons	n_F	N_F	η_F
c	3	3	0	e	8	0	30/24
u	7	3	1/2	μ	4	0	17/24
b	2	1	1/4	τ	3	0	11/24
s	4	0	3/4	ν_e	16	3	2
d	6	0	13/12	ν_μ	9	3	1
				ν_τ	6	3	1/2

8. RUNNING MASSES OF THE FERMIONS

The exact one loop solution of the RGE for change in mass values with energy is given by equation(2.16). With $Z_F(t) = 8\pi^2 \frac{d}{dt} \log B_F^2(t)$, this equation reduces to

$$\frac{M_{top}^2}{M_F^2(t)} = \frac{M_{top}^2}{M_U^2}\frac{B_F^2(t_X)}{B_F^2(t)} + \frac{A_F}{8\pi^2}\int_t^{t_X} dt_1 \frac{B_F^2(t_1)}{B_F^2(t)}, \tag{8.1}$$

and

$$M_F(t) = M_F(M_X)\frac{B_F(t)/B_F(t_X)}{\left(1 + \frac{M_U^2}{M_{top}^2}\frac{A_F}{8\pi^2}\int_t^{t_X} dt_1 \frac{B_F^2(t_1)}{B_F^2(t_X)}\right)^{1/2}}. \tag{8.2}$$

If we take

$$B_F(t) = \lambda^{n_F(1-t/t_X)} = \exp\left(n_F(1-t/t_X)\log_e \lambda\right), \tag{8.3}$$

and

$$M_F(t) = M_U \lambda^{n_F}\left(1 + \frac{M_U^2}{M_{top}^2}\frac{A_F}{8\pi^2}\int_t^{t_X} dt_1 \lambda^{2n_F(1-t_1/t_X)}\right)^{1/2}, \tag{8.4}$$

with

$$\begin{aligned}\int_t^{t_X} dt_1 \lambda^{-2n_F t_1/t_X} &= \int_t^{t_X} dt_1 e^{-2n_F t_1/t_X \log_e \lambda} \\ &= -\frac{t_X}{2n_F}\frac{1}{\log_e \lambda}\left[e^{-2n_F \log_e \lambda} - e^{-2n_F t/t_X \log_e \lambda}\right] \\ &= -\frac{t_X}{2n_F}\frac{1}{\log_e \lambda}\left[\lambda^{-2n_F} - \lambda^{-2n_F t/t_X}\right],\end{aligned} \tag{8.5}$$

we have, from equation (8.2)

$$M_F(t) = M_F(M_X)\frac{\lambda^{n_F(1-t/t_X)}}{\left[1+\frac{M_F^2(M_X)}{M_{top}^2}\frac{A_F}{8\pi^2}\lambda^{2n_F}\lambda^{2n_F/t_X}\left(\lambda^{-2n_F}-\lambda^{-2n_F t/t_X}\right)\right]^{1/2}} \tag{8.6}$$

We shall not use this for constructing the tables. A simpler approximate form ignoring A_F terms of equation (8.6),

$$M_F(t) = M_F(M_X)\lambda^{n_F\left(1-\frac{t}{33}\right)} \tag{8.7}$$

is presented in tabular form in Table-V to Table-VII. This gives the same CKM matrix. Now, we present our results in the following tables.

TABLE V: Variation of $M_{top}(t)$ with t= $log_e(\mu/M_Z)$

t	0	3	6	9	12	15	18	21	24	27	30	33
$M_{top}(t)$(GeV)	175.88	166.98	159.60	153.25	147.61	142.46	137.64	133.02	128. 51	124.51	119.56	115.00

From the above values, we find that $\frac{d^2 M_{top}(t)}{dt^2}$ is nearly zero. The top mass variation is approximately

$$M_{top}(t) = \left[175. - \frac{t}{33}\times 60\right] \ \texttt{GeV}. \tag{8.8}$$

TABLE VI: Running Lepton Masses in GeV, t_X=33.

t/t_X	M_e	M_μ	M_τ	M_{ν_e}	M_{ν_μ}	M_{ν_τ}
0	0.00051	0.105	1.77	$3{\cdot}10^{-9}$	$1.9{\cdot}10^{-4}$	0.018
0.1	0.00175	0.2115	2.687	$3.43{\cdot}10^{-8}$	$7.19{\cdot}10^{-4}$	0.043
0.2	0.006	0.426	4.079	$3.92{\cdot}10^{-7}$	$2.72{\cdot}10^{-3}$	0.104
0.3	0.0206	0.8575	6.192	$4.49{\cdot}10^{-6}$	$1.03{\cdot}10^{-2}$	0.25
0.4	0.02	1.726	9.399	$5.13{\cdot}10^{-5}$	$3.9{\cdot}10^{-2}$	0.6
0.5	0.242	3.476	14.267	$5.87{\cdot}10^{-4}$	0.1478	1.44
0.6	0.830	6.999	21.658	$6.72{\cdot}10^{-3}$	0.56	3.46
0.7	2.849	14.081	32.876	$7.68{\cdot}10^{-2}$	0.01	8.3
0.8	9.774	28.369	49.910	0.482	8.02	19.93
0.9	33.527	57.118	75.758	10.05	30.374	47.9
1.0	115.0	115.0	115.0	115.0	115.0	115.0

TABLE VII: Values of Running quark masses in GeV (other than the top quark), t_X=33.

t/t_X	M_c	M_u	M_b	M_s	M_d
0.0	1.27	0.0042	4.23	0.159	0.0075
0.1	2.62	0.0116	5.88	0.307	0.0196
0.2	3.12	0.0324	8.18	0.593	0.051
0.3	4.9	0.09	11.39	1.146	0.135
0.4	7.7	0.25	15.85	2.213	0.354
0.5	12.0	0.695	22.05	4.276	0.9287
0.6	18.96	1.9307	30.687	8.26	2.434
0.7	34.16	5.3636	42.696	15.95	6.383
0.8	46.69	14.9	59.406	30.82	16.73
0.9	73.28	41.395	82.651	59.59	43.86
1.0	115.0	115.0	115.0	115.0	115.0

The graphs are much more revealing. The variation of mass for the Up quarks, including the top, have been shown in figure-1, for the Down in figure-2 , for the Electrons in figure-3 and for the Neutrinoes in figure- 4. All have been compressed in figure-5 to allow a glance at the totality of the descent and ascent of the masses to the unification mass of 115 GeV. We believe that an exact analysis will not differ much from those presented in these figures.

9. CONCLUDING REMARKS

Deo and Maharana[14] have already given exact solutions for the one loop RGEs in MSSM. They proved that all the fermions might have originated from a common mass $M_U \simeq 115$ GeV at the GUT energy of $M_X \simeq 2.2 \times 10^{16}$ GeV in a perturbative scheme. This has not been confirmed by authors working in this field as they have not looked at the solutions which proved the same wayas one loop gauge sector renormalisation group equation.

As the energy diminishes, the mass of the top increases to 175 GeV at the Z-meson mass M_Z, whereas, masses of all other quarks and tiny leptonic masses increase from their value at M_Z to M_U of each 115 GeV. Hopefully, the exact nature of this variation could be obtained from RGE. By introducing an auxiliary function $B(t)$ through the mixing terms of RGE, a very simplified expression can be obtained for both the mass values at $M_Z(t = 0)$ and their variation till M_X when, the masses of the fermions which keep changing, attain the final value at M_X.

The success of the analysis of the Wolfenstein and the ratio parametrization of RRR given by equation(1.31) is clearly brought out by our solutions. An extension to running masses at linearity level, leads to a very simple formula for the fermions other than the top,

$$M_F(t) = M_U \left(\frac{M_F}{M_U}\right)^{(1-\frac{t}{t_X})} = M_U\ \lambda^{n_F(1-\frac{t}{t_X})}, \tag{9.1}$$

where $M_F^{exp.} = M_F(M_Z) = \lambda^{n_F}$. The increase is exponential. Much more exact analytic studies are needed to deduce the values of the n_F from RGE. As a first step, we follow up the texture analysis procedure by introducing a similar, but not the same function $B(t)$[13], which is such that $d \log B(t) = d \log M_{top}(t)$. The resulting non-uniqueness is taken advantage of. Introducing known gauge couplings and the values of n_F, the effect of gauge interaction have been calculated.

In the foregoing sections 1 to 8, there has been two important omissions. We have not remarked neither about higher loop effects nor about the threshhold corrections. They may be very large due to very heavy top. There has been no mention of the CKM phases. But, we work with three generations only. So, in this case there is precisely, one phase angle ϕ. This is defined from the Wolfenstein parametrisation as

$$\lambda = \left(\frac{M_d}{M_s} + \frac{M_u}{M_c} + 2\sqrt{\frac{M_d}{M_s}\frac{M_u}{M_c}}\cos\phi\right)^{\frac{1}{2}}, \tag{9.2}$$

and can be deduced by extending the Gatto-Sarton-Tonio-Oakes (GSTO)[18] to next leading order of solution of RGE, $\lambda = (M_d/M_s)^{1/2}$. Putting in the numbers n_F from table-III, we get

$$\cos\phi \simeq -\frac{\lambda}{2} \simeq -0.1, \qquad \phi \simeq 95^0. \tag{9.3}$$

This result is reasonable.

It may be argued that there are too many parameters still in the guise of the rotation integers n_F even though λ could be determined. Let us recall the case of the unification of the strong, the weak, the electromagnetic and the gravitational interactions. One consciously omits gravity and lists the other three in order of their strengths. Here the top mass, with the largest value,has been deduced from the knowledge of the unification mass and gauge couplings quite accurately. For F=2, the two successive rotations i.e., 0 to 4π gives n_F=2 and $M_U\lambda^2$ falls near the mass of the bottom. This process of complete successive rotations yields the masses of all the quarks and electrons, except n_F=5. However, there is a GUT prediction by Georgi and Jarlskog[3], that the ratio $M_s/M_d \simeq 25 \simeq \lambda^{-2}$. This may be the reason why n_F jumps from 4 to 6. There is also another accurate prediction

$$\frac{M_d/M_s}{(1 - M_d/M_s)^2} = 9.\frac{M_e/M_\mu}{(1 - M_e/M_\mu)^2}. \tag{9.4}$$

With these GUT supplements, there is no other unknown additional parameter left. The unification mass $M_U \simeq 115$ GeV is the only input.

Corroborating and in consonance with a lot of excellent work in neutrino physics have been done by many workers ,e.g. Babu, Mohapatra and Barr[20]. The neutrino masses fall with the rotational integers, determined for the neutrinoes in the table-V in a very nice way.

The figure-5 illustrates the significance of this approach of solving the RGE. The similarity of figure-5 with gauge unification graph is striking. Thus, we prove, beyond doubt that the original ideas of Pati and Salam[1] is correct as well as the SUSY standard model. We have given many extrapolatory ideas which can be successfully implemented so that SUSY standard model world can be characterised by only three parameters, the GUT mass $M_X \simeq 2.2 \times 10^{16}$, the GUT coupling strength $\alpha_{GUT} \simeq \frac{1}{24.5}$ and common-origin -fermion mass $M_U \simeq 115$ GeV. This is a simplification beyond expectation and very much gratifying.

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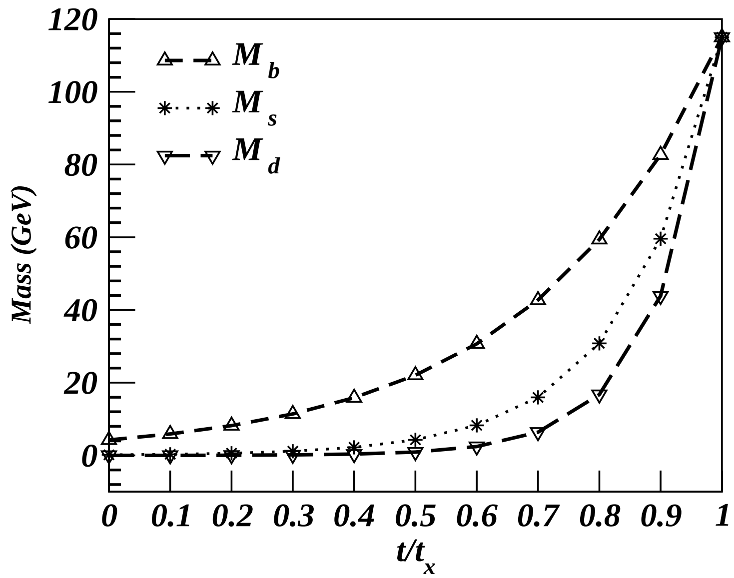

FIG. 2: Variation of masses of Down-quarks in Gev with t/t_X , t=log(μ/M_Z) and t_X=33.

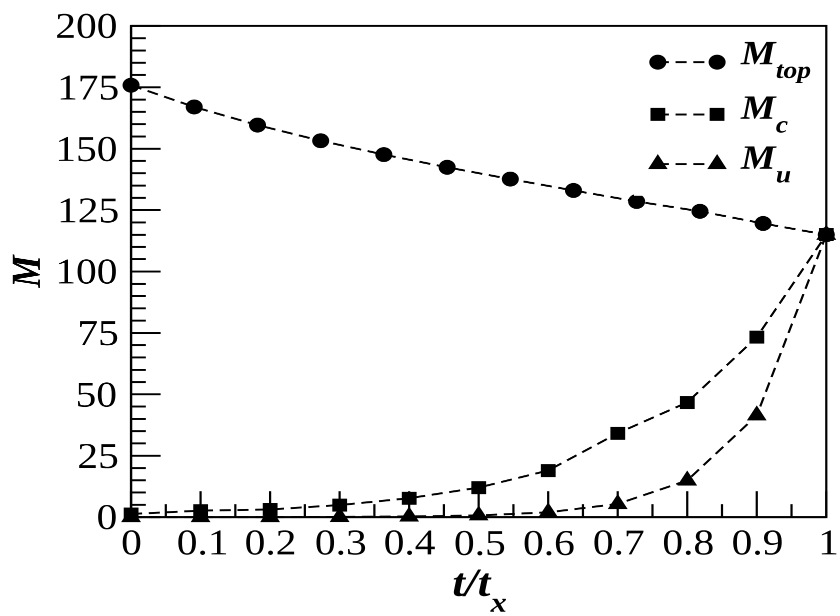

FIG. 1: Variation of masses of Up-quarks in Gev with t/t_X , t=log(μ/M_Z) and t_X=33.

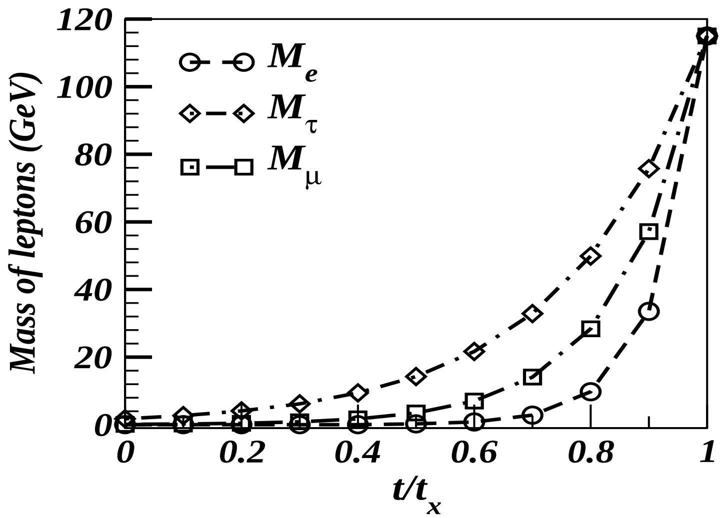

FIG. 3: Variation of masses of Electrons in Gev with t/t_X , t=log(μ/M_Z) and t_X=33.

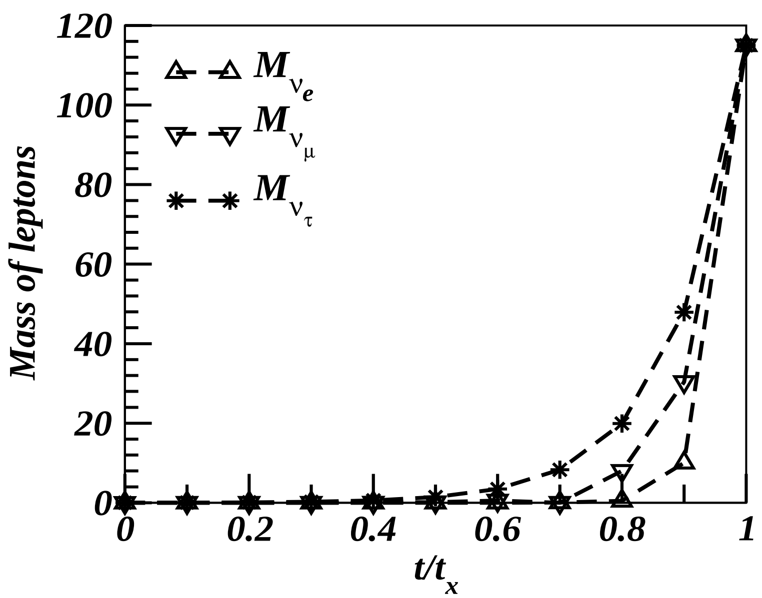

FIG. 4: Variation of masses of Neutrinos in Gev with t/t_X , t=log(μ/M_Z) and t_X=33.

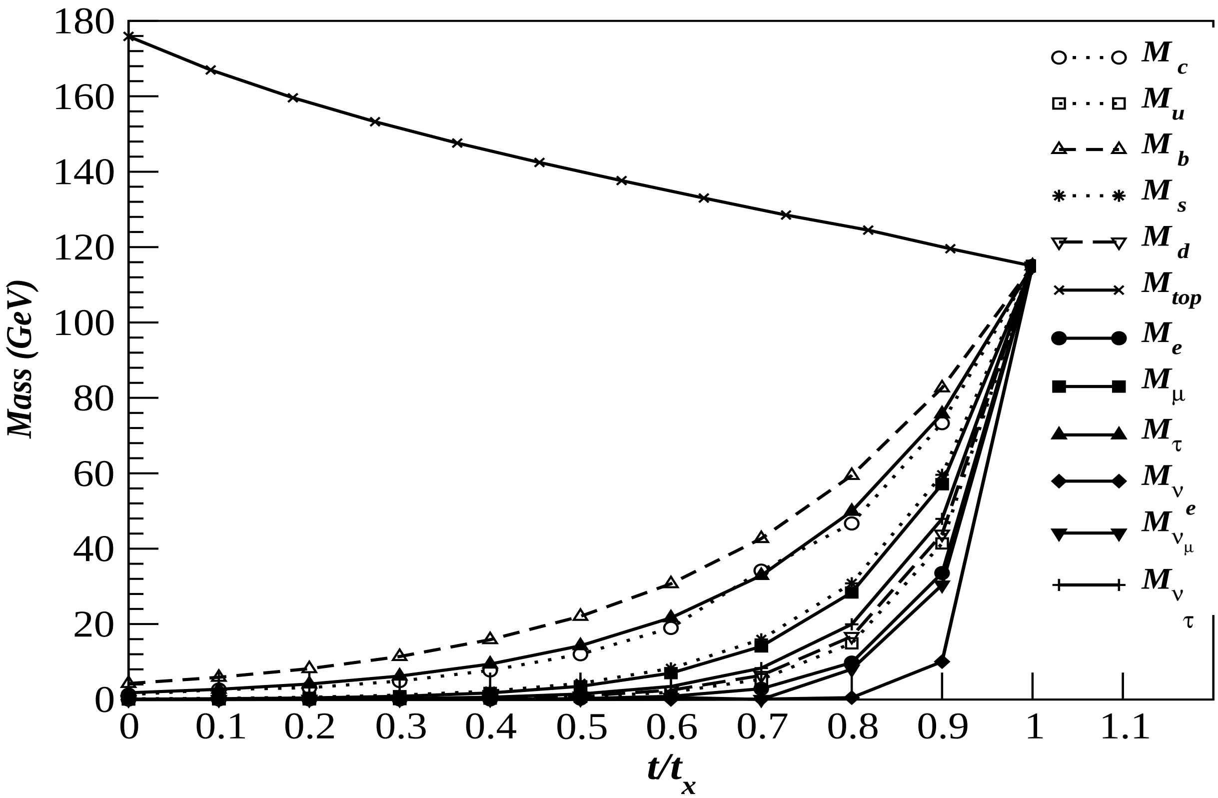

FIG. 5: Variation of masses of all 12 fermions in Gev with t/t_X , t=log(μ/M_Z) and t_X=33.